Linear Algebra, Winter 2022

List 7

More complex numbers, intro to polynomials

164. Re-write $(r\,e^{i\theta})^3$ in the form $\_\_\,e^{—i}$. $\boxed{r^3\,e^{3\theta\,i}}$

$$e^{\theta i} = \cos(\theta) + i\sin(\theta)$$

165. Re-write $10e^{(\pi/4)i}$ in the form $\_\_ + \_\_i$.
$10(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = 10(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = \boxed{5\sqrt{2} + 5\sqrt{2}i}$

166. Re-write $(2e^{7i})^{10}$ in the form $\_\_ + \_\_i$.

$$\boxed{1024\cos(70) + 1024\sin(70)i} \text{ or } 648.52 + 792.46i$$

(Note: $\cos(70) \approx 0.633$ is **not** the same as $\cos(70°) \approx 0.342$.)

167. Re-write $-\sqrt{5} + \sqrt{15}\,i$ in the form $\_\_e^{—i}$. Using **Task 158(b)**, $\boxed{\sqrt{20}e^{(3\pi/4)i}}$

168. If z is a complex number with $|z| = 4$, what is $|z^2|$? $\boxed{16}$

169. If z is a complex number with $\arg(z) = 5\pi/6$, what is $\arg(z^2)$? The angle is $\frac{5\pi}{3}$, but arguments are usually given in the interval $(-\pi, \pi]$, so this is $\boxed{\frac{-\pi}{3}}$.

170. If w is a complex number with $\arg(w) = \pi/10$, what is $\arg(w^{446})$? $\boxed{\frac{3\pi}{5}}$

171. Write $\left(\dfrac{\sqrt{3}-i}{1+i}\right)^6$ in the form $a + bi$. $\boxed{-8i}$
(Hint: $\sqrt{3} - i = 2e^{(-\pi/6)i}$ and $1 + i = \sqrt{2}e^{(\pi/4)i}$.)

Rectangular form: $a + b\,i$, or $a + i\,b$, or $b\,i + a$, or similar.
Polar form: $r\big(\cos\theta + i\sin\theta\big)$, or $r\cos(\theta) + r\sin(\theta)\,i$, or similar. Requires $r \geq 0$.
Exponential form: $r\,e^{\theta\,i}$, or $r\,e^{i\,\theta}$.

172. Write the following in rectangular form.

(a) $e^{\frac{\pi}{4}i}$ $\boxed{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i}$ or $\boxed{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$

(b) $2e^{i\pi/6}$ $\boxed{\sqrt{3} + i}$

(c) $5e^{-i\pi/3}$ $\boxed{\frac{5}{2} - \frac{5\sqrt{3}}{2}i}$

(d) $-8e^{\pi i}$ $\boxed{8}$ or $\boxed{8 + 0i}$

(e) $\sqrt{9} + \sqrt{-9}$ $\boxed{3 + 3i}$

173. For $z = 1 + i$ and $w = 3e^{(\pi/4)i}$, calculate the following. For complex values, you may give the answer in rectangular or polar or exponential form (your choice).

(a) $|w|$ $\boxed{3}$

(b) $|zw|$ $\boxed{3\sqrt{2}}$

(c) $|z/w|$ $\boxed{\sqrt{2}/3}$

(d) $-w$ $\boxed{3e^{(5\pi/4)i}}$

(e) $(\bar{w})^2$ $\boxed{-9i}$ or $\boxed{9e^{(-\pi/2)i}}$

(f) zw $\boxed{3\sqrt{2}i}$ or $\boxed{3\sqrt{2}e^{(\pi/2)i}}$

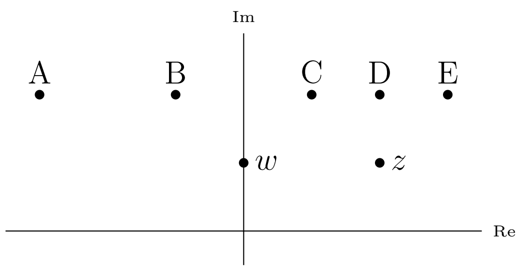
(g) $z + w$ $\boxed{(1 + \frac{\sqrt{3}}{2}) + (1 + \frac{\sqrt{3}}{2})i}$ or $\boxed{(3 + \sqrt{2})e^{(\pi/4)i}}$

(h) $z + z^2$ $\boxed{1 + 3i}$

(i) z/w $\boxed{\sqrt{2}/3}$

(j) w/z $\boxed{3/\sqrt{2}}$

174. Which of the points A-E below could be $z + w$? $\boxed{\text{D}}$



175. Which of the points A-E above could be zw? $\boxed{\text{B}}$

176. Write $(5e^{70°i})(2e^{-40°i})$ in exponential form and polar form and rectangular form.

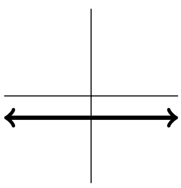
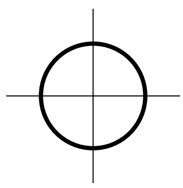
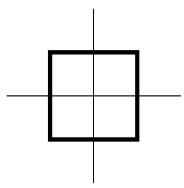
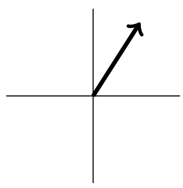
Exp: $\boxed{10e^{30°i}}$ Polar: $\boxed{10\cos(30°i) + 10\sin(30°i)}$ Rect: $\boxed{5\sqrt{3} + 5i}$

177. Write $(3 + 2i)(3 - 2i)$ in rectangular form. $\boxed{13}$

☆178. Which of the following is equal to i^i? $\boxed{\text{(C)}}$

(A) $\dfrac{i}{\sqrt{2}}$ (B) $\ln(2) + i$ (C) $\dfrac{1}{\sqrt{e^{\pi}}}$ (D) $e^{\sqrt{3}}$ (E) $2\pi i$ (F) $\dfrac{\ln(\pi)}{2}$

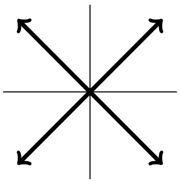
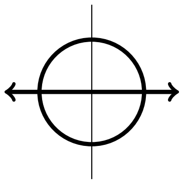
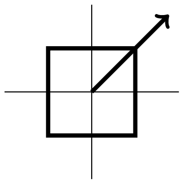
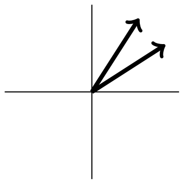
179. Which of the following shows all complex numbers for which $|z| = 1$? $\boxed{\text{(B)}}$

(A)  (B)  (C)  (D) 

180. Which of the images from #179 shows all complex numbers with $\arg(z) = 1$? $\boxed{\text{(D)}}$

181. Which of the images from #179 shows all complex numbers for which $z + i$ is real (meaning that the imaginary part of $z + i$ is zero)? $\boxed{\text{(A)}}$

☆182. Which of the following shows all complex numbers for which $\dfrac{1}{1+z^2}$ is real? $\boxed{\text{(B)}}$

(A)  (B)  (C)  (D) 

The **conjugate** of the complex number z, written as $\bar{z}$ and spoken as "Z bar", is the reflection of z over the real-axis. In formulas,

$$\overline{a+bi} = a - bi \qquad \text{and} \qquad \overline{re^{\theta i}} = re^{-\theta i}$$

if a, b, r, θ are real numbers.

183. Given that $\bar{z} = 5 + 2i$ and $\overline{w} = 3 - 6i$, calculate $\overline{w + z}$. $\boxed{8 - 4i}$

184. (a) For $z = \dfrac{\sqrt{7}}{2} + \dfrac{\sqrt{11}}{3}i$, calculate $z + \bar{z}$. $\boxed{\sqrt{7}}$

(b) For $z = 31 + \dfrac{\sqrt{3+\pi}}{\log(4) - 12}i$, calculate $z + \bar{z}$. $\boxed{\sqrt{62}}$

(c) For $z = 9e^{(\pi/8)i}$, calculate $z \cdot \bar{z}$. $\boxed{81}$

(d) For $z = \sqrt{26}e^{(8e^3 - \sqrt{5})i}$, calculate $z \cdot \bar{z}$. $\boxed{26}$

185. How are $|z|$ and $|\bar{z}|$ related? equal: $\boxed{|\bar{z}| = |z|}$ How are $\arg(z)$ and $\arg(\bar{z})$ related? negatives: $\boxed{\arg \bar{z} = -\arg z}$

186. (a) Give an example of a number z for which $z + \bar{z} = -12$, or explain why no such z can exist. $z = -6 + bi$ for any real number b.

(b) Give an example of a number z for which $z \cdot \bar{z} = -12$, or explain why no such z can exist. Can't exist because $z \cdot \bar{z} = |z|^2$ is always positive.